

Olympiad Problems Set -02

20 Selected Problems with Answer Key

Questions

1. For how many integers is the number $x^4 - 51x^2 + 50$ negative?
2. Find an integer x that satisfies $x^5 - 101x^3 - 999x^2 + 100900 = 0$.
3. Let $f(x) = \frac{x^{2010}}{x^{2010} + (1-x)^{2010}}$. Find the value of $\frac{f\left(\frac{1}{2011}\right) + f\left(\frac{2}{2011}\right) + \cdots + f\left(\frac{2010}{2011}\right)}{15}$.
4. Given that $a + \frac{1}{a+1} = b + \frac{1}{b-1} - 2$ and $a - b + 2 \neq 0$, find the value of $ab - a + b$.
5. Let n be a positive integer such that one of the roots of $4x^2 - (4\sqrt{3} + 4)x + \sqrt{3}n - 24 = 0$ is an integer. Find the value of n .
6. If p, q and r are distinct roots of $x^3 - x^2 + x - 2 = 0$, then $p^3 + q^3 + r^3$ equals
7. If x, y and z are positive integers such that $27x + 28y + 29z = 363$, find the value of $(x + y + z)$.
8. Given that $(m - 2)$ is a positive integer and it is also a factor of $3m^2 - 2m + 10$. Find the sum of all such values of m .
9. Let $g(x) = x^5 + x^4 + x^3 + x^2 + x + 1$. What is the remainder when $g(x^{12})$ is divided by $g(x)$?
10. There are n balls in a box, numbered $1, 2, 3, \dots, n$. One ball is removed, and the sum of the numbers on the remaining balls is 5048. If the number on the removed ball is m , find the value of m .
11. Let x, y, z be three real numbers such that $xy + yz + xz = 4$. Find the least possible value of $x^2 + y^2 + z^2$.
12. If $y + 4 = (x - 2)^2$, $x + 4 = (y - 2)^2$, and $x \neq y$, what is the value of $x^2 + y^2$?
13. Let a, b and c be the lengths of the three sides of a triangle. Suppose a and b are the roots of $x^2 + 4(c + 2) = (c + 4)x$, and the largest angle of the triangle is x° . Find the value of x .
14. Find the number of pairs (m, n) of integers which satisfy $m^3 + 6m^2 + 5m = 27n^3 + 9n^2 + 9n + 1$.
15. Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b . Find the value of $(a - b)$.
16. It is given that $x = \frac{1}{2 - \sqrt{3}}$. Find the value of $x^6 - 2\sqrt{3}x^5 - x^4 + x^3 - 4x^2 + 2x - \sqrt{3}$.
17. Given that a, b are roots of $x^2 - 7x + 8 = 0$, where $a > b$. Find the value of $\frac{2}{a} + 3b^2$ without solving the equation.

18. Suppose that the two roots of $\frac{1}{x^2 - 10x - 29} + \frac{1}{x^2 - 10x - 45} - \frac{2}{x^2 - 10x - 69} = 0$ are α and β . Find the value of $(-\alpha\beta)$.
19. If the equation in x has real roots, find the values of a and b : $x^2 + 2(1 + a)x + (3a^2 + 4ab + 4b^2 + 2) = 0$.
20. The total number of integers between 0 and 10^5 having digit sum equal to 8 is n . Then find $5(500 - n)$.

Answer Key

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|-------------|-----------------------------------|
| 1. 12 | 11. 4 |
| 2. $x = 10$ | 12. 15 |
| 3. 67 | 13. $x = 90$ |
| 4. 2 | 14. 0 |
| 5. $n = 12$ | 15. 3 |
| 6. 4 | 16. 2 |
| 7. 13 | 17. $\frac{403 - 85\sqrt{17}}{8}$ |
| 8. 51 | 18. 39 |
| 9. 6 | 19. $a = 1, b = -\frac{1}{2}$ |
| 10. $m = 2$ | 20. 25 |