Olympiad Problems Set -01

20 Selected Problems with Answer Key

Questions

1. Let p be a real number such that the equation $2y^2 - 8y = p$ has only one solution. Then find the value of $p + 64$.

2. If $|x| + x + 5y = 2$ and $|y| - y + x = 7$, find the value of $(x + y)^3$.

3. Define a function on the positive integers recursively by $f(1) = 2$, $f(n) = f(n-1) + 1$ if n is even, and $f(n) = f(n-2) + 2$ if n is odd and greater than 1. What is $f(2017) - 2000$?

4. Let x and y be real numbers such that $x^2 + y^2 = 2x - 2y + 2$. What is the largest possible value of $x^2 + y^2 - \sqrt{32}$?

5. Suppose that $a + x^2 = 2006$, $b + x^2 = 2007$, $c + x^2 = 2008$ and $abc = 3$. Find $\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab} - \frac{1}{a} - \frac{1}{b} - \frac{1}{c}$.

6. Given that x and y are real numbers satisfying $x + xy + y = 2 + 3\sqrt{2}$ and $x^2 + y^2 = 6$, the value of $|x + y + 1| = a + \sqrt{b}$. Find the product of a and b.

7. Suppose that x and y are nonzero real numbers such that $\frac{3x + y}{x - 3y} = -2$. What is the value of $\left(\frac{x + 3y}{3x - y}\right)^4$?

8. It is known that there is only one pair of positive integers a, b such that $a \leq b$ and $a^2 + b^2 + 8ab = 2010$. Find $a + b$.

9. Given that α and β are the real roots of $x^2 - 2x - 1 = 0$, find the value of $5\alpha^4 + 12\beta^3$.

10. Given that x and y are both negative integers satisfying $y = \frac{10x}{10 - x}$, find the maximum value of $y + 20$.

11. If two positive integers m and n, both bigger than 1, satisfy $2005^2 + m^2 = 2004^2 + n^2$, find the value of $m + n - 200$.

12. Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

13. For what values of b do the equations $1988x^2 + bx + 8891 = 0$ and $8891x^2 + bx + 1988 = 0$ have a common root?

14. Let a and b be the roots of $x^2 - mx + 2 = 0$. Suppose that $a + \frac{1}{b}$ and $b + \frac{1}{a}$ are the roots of $x^2 - px + q = 0$. What is $2q$?

15. What is the sum of all the roots of the equation $\sqrt{5|x|+8} = \sqrt{x^2-16}$?

16. Let a, b, c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ is

17. Find the value of $a + b$ where $\dfrac{1}{a!} - \dfrac{1}{b!} = \dfrac{3}{1!+2!+3!} + \dfrac{4}{2!+3!+4!} + \cdots + \dfrac{22}{20!+21!+22!}$.

18. Given that $y = (x-16)(x-14)(x+14)(x+16)$, find the maximum value of $(-0.1y)$.

19. The zeroes of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of all possible values of a?

20. If $x = \dfrac{a}{b+c} = \dfrac{b}{a+c} = \dfrac{c}{a+b}$, then find the value of x.

Answer Key

1. 56
2. 27
3. 18
4. $2 - 4\sqrt{2}$
5. 1
6. $ab = 6$
7. 16
8. 42
9. 169
10. 15
11. 11
12. 1 (namely $k = 122$)
13. $b = \pm 10879$
14. 9
15. 0
16. 1
17. 24
18. 90
19. 16
20. $x = \frac{1}{2}$ or $x = -1$