

JEE Mathematics

The Last-Mile Formula & Trap Sheet

Not every formula — only the ones that decide marks. Built for revision under time pressure.

A note before you start using this sheet:

This is a **deliberately curated** sheet, not a complete formula encyclopedia. Every standard JEE formula book already gives you 300+ formulas in small print — and in the last few days before an exam, that volume works against you: you re-read things you already know cold, and skim past the traps that actually cost marks. This sheet leaves out routine/low-yield formulas (ones you already recall instantly, or that rarely decide a question on their own) on purpose, so it stays something you can read start to finish in one sitting instead of a reference you passively scroll.

What is included is chosen for **high-frequency, high-mistake-rate** value: formulas that appear repeatedly across JEE Main/Advanced papers, paired with a **TRAP/TRICK** box per topic that flags the specific sign error, missing condition, or wrong assumption that typically costs the mark — not the formula itself. Some entries (e.g. location of roots, angle bisector sign convention, inverse-trig sum conditions) genuinely need this extra observation to apply correctly; that is the point of the box, not an afterthought.

Use this alongside your full formula book or class notes, not instead of them. Treat this as the “final revision + error-check” layer on top of complete preparation, not as your primary or sole source. If a topic here feels thin, that is a signal to go back to the complete notes for that chapter — it is not a claim that nothing else in that chapter matters.

1. Quadratics & Theory of Equations

Core Results

- $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{D}}{2a}, D = b^2 - 4ac$
- $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$; equation with roots α, β : $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
- $D > 0$ real distinct; $D = 0$ equal; $D < 0$ complex conjugates
- Common root of $a_1x^2 + b_1x + c_1 = 0, a_2x^2 + b_2x + c_2 = 0$: use cross-multiplication with $a_1b_2 - a_2b_1$ etc.
- Location of roots (for $a > 0$): both $> k \Rightarrow D \geq 0, af(k) > 0, -b/2a > k$. Root between $(k_1, k_2) \Rightarrow f(k_1) \cdot f(k_2) < 0$.

TRAP / TRICK

Sign of a matters: location-of-roots conditions use $a \cdot f(k) > 0$, not $f(k) > 0$ — flips if $a < 0$.

“Roots real” $\neq$ “ $D \geq 0$ alone” if coefficients depend on a parameter — always re-check $a \neq 0$.

Common trap: students forget the equation may degenerate to linear if $a = 0$ in a parametric family — check that case separately.

2. Sequences & Series

AP / GP / HP

- AP: $t_n = a + (n-1)d, S_n = \frac{n}{2}[2a + (n-1)d]$
- GP: $t_n = ar^{n-1}, S_n = \frac{a(r^n - 1)}{r - 1}$ ($r \neq 1$), $S_\infty = \frac{a}{1-r}$, $|r| < 1$
- AM $\geq$ GM $\geq$ HM, equality iff all terms equal
- $\sum r = \frac{n(n+1)}{2}, \sum r^2 = \frac{n(n+1)(2n+1)}{6}, \sum r^3 = \left(\frac{n(n+1)}{2}\right)^2$

TRAP / TRICK

S_∞ formula needs $|r| < 1$ — do not apply blindly; check convergence first, JEE loves planting $r \geq 1$ GPs.

Telescoping series: write t_r as $f(r) - f(r+1)$ before summing — saves huge time on $\sum \frac{1}{r(r+1)}$ -type.

AM-GM equality trap: when a question says “minimum value,” check if equality is actually attainable in the given domain.

3. Binomial Theorem

Expansion & Terms

- $(a+b)^n = \sum_{r=0}^n {}^nC_r a^{n-r} b^r$; general term $T_{r+1} = {}^nC_r a^{n-r} b^r$

- Middle term: n even $\Rightarrow (\frac{n}{2} + 1)$ th term; n odd $\Rightarrow$ two middle terms
- Greatest term: ratio $\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r} \cdot \frac{b}{a} \geq 1$ gives the greatest term's index
- ${}^nC_0 + {}^nC_1 + \dots + {}^nC_n = 2^n$; sum of odd-position = sum of even-position = 2^{n-1}
- $(1+x)^n \approx 1 + nx$ for $|x| \ll 1$ (approximation questions)

TRAP / TRICK

Independent-of- x term: set the power of x in T_{r+1} to zero and solve for r — verify r is a non-negative integer $\leq n$, else “no such term.”

$(\sqrt{A} + B)^n$ integer-part problems: use $(1+f)(1-f) = k^n$ trick — always check whether n is odd/even before applying.

Largest binomial coefficient is at $r = n/2$ (even n) — don't confuse with largest term (which depends on a, b).

4. Permutations & Combinations

Selection / Arrangement

- ${}^nP_r = \frac{n!}{(n-r)!}, {}^nC_r = \frac{n!}{r!(n-r)!}$; circular arrangements: $(n-1)!$, if clockwise=anticlockwise treated same: $\frac{(n-1)!}{2}$
- At least one of n distinct objects selected: $2^n - 1$
- Distinct objects into groups (identical boxes) / distinguishable boxes — always ask “are boxes distinct?” first
- Divisors of $N = p^a q^b r^c$: number = $(a+1)(b+1)(c+1)$; sum via GP product
- Non-negative integer solutions of $x_1 + \dots + x_k = n$: ${}^{n+k-1}C_{k-1}$; positive solutions: ${}^{n-1}C_{k-1}$

TRAP / TRICK

Identical items: arrangements of n items with repetitions use $\frac{n!}{p!q!r!}$ — forgetting repeated-letter division is the #1 P&C error.

“At least”/“at most” — always compute via complement (total — unwanted) rather than direct casework; far fewer errors.

Circular table with distinguishable seats (numbered chairs) is just linear arrangement $n!$, NOT $(n-1)!$ — read the question carefully.

5. Probability

Core Rules

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A/B) = \frac{P(A \cap B)}{P(B)}$; independence: $P(A \cap B) = P(A)P(B)$

- Bayes: $P(B_i/A) = \frac{P(B_i)P(A/B_i)}{\sum P(B_j)P(A/B_j)}$
- Binomial: $P(r) = {}^nC_r p^r q^{n-r}$, mean = np , variance = npq

⚡ TRAP / TRICK

Independent $\neq$ mutually exclusive – mixing these up is the most common conceptual slip; mutually exclusive events with nonzero probability are never independent.

Conditional probability word problems: identify exactly what's "given" before writing $P(A/B)$ – reversing A, B is a classic trap in Bayes' questions.

"Without replacement" changes probabilities each draw – always re-check sample space size after each draw.

6. Complex Numbers

Algebra & Geometry

- $|z_1 z_2| = |z_1| |z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2 + 2k\pi$
- Cube roots of unity: $1, \omega, \omega^2$; $1 + \omega + \omega^2 = 0$, $\omega^3 = 1$
- $|z - z_1| = |z - z_2| \Rightarrow$ perpendicular bisector of $z_1 z_2$
- $\left| \frac{z - z_1}{z - z_2} \right| = k$ ($k \neq 1$) $\Rightarrow$ circle (Apollonius); $k = 1 \Rightarrow$ straight line
- Rotation: $\frac{z_3 - z_2}{z_1 - z_2} = \frac{z_3 - z_2}{z_1 - z_2} e^{i\theta}$

⚡ TRAP / TRICK

$\arg(z_1/z_2)$ **principal value:** adding/subtracting args can throw the result outside $(-\pi, \pi]$ – always adjust by $\pm 2\pi$ at the end.

Locus questions: translate $|z - z_1| + |z - z_2| = k$ carefully – if $k = |z_1 - z_2|$ it's a *segment*, not an ellipse (degenerate case often missed).

ω -power problems: reduce exponents mod 3 first ($\omega^{3n+r} = \omega^r$) before simplifying – saves algebra and avoids sign slips.

7. Matrices & Determinants

Key Identities

- $|kA| = k^n |A|$ for $n \times n$; $|AB| = |A||B|$; $|A^{-1}| = 1/|A|$
- $A^{-1} = \frac{1}{|A|} \text{adj}(A)$; exists iff $|A| \neq 0$
- Symmetric: $A = A^T$; Skew-symmetric: $A^T = -A$ (diagonal entries = 0)
- System $AX = B$: unique solution if $|A| \neq 0$; if $|A| = 0$, infinite/no solution depending on $(\text{adj}A)B$
- $\text{adj}(AB) = \text{adj}(B) \text{adj}(A)$; $|\text{adj} A| = |A|^{n-1}$

⚡ TRAP / TRICK

Row/column operations must be applied one at a time in determinant simplification – doing two simultaneous row ops can silently change the value.

$|A| = 0$ **case:** don't assume "no solution" – check consistency via $(\text{adj}A)B$; could be infinite solutions.

Matrix multiplication is not commutative – $AB \neq BA$ in general; never cancel matrices like scalars.

8. Trigonometric Ratios & Equations

Formulas

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$; $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$; $\cos 2A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1$
- $\sin 3A = 3 \sin A - 4 \sin^3 A$; $\cos 3A = 4 \cos^3 A - 3 \cos A$
- $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$ (and 3 similar identities)
- General solutions: $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha$; $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$; $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha$

⚡ TRAP / TRICK

Squaring trig equations introduces extraneous roots – always verify each solution in the original equation.

Dividing by $\cos \theta$ or $\sin \theta$ silently loses solutions where that factor is zero – check those separately first.

Domain restriction after substitution (e.g. $t = \tan \theta/2$) excludes $\theta = \pi$ – always test boundary values separately.

9. Inverse Trigonometric Functions

Principal Values & Identities

- $\sin^{-1} x \in [-\pi/2, \pi/2]$, $\cos^{-1} x \in [0, \pi]$, $\tan^{-1} x \in (-\pi/2, \pi/2)$
- $\sin^{-1} x + \cos^{-1} x = \pi/2$; $\tan^{-1} x + \cot^{-1} x = \pi/2$
- $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$ *only if* $xy < 1$; add π if $xy > 1$, $x, y > 0$
- $2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2}$ valid only for $|x| \leq 1$ (piecewise otherwise)

⚡ TRAP / TRICK

Range mismatch is the #1 error: $\sin^{-1}(\sin \theta) \neq \theta$ outside $[-\pi/2, \pi/2]$ – always reduce θ into the principal range first.

Sum formulas are piecewise – check the $xy < 1$ vs $xy > 1$ condition before applying; JEE frequently tests exactly this boundary.

Never assume domain $x \in \mathbb{R}$ for identities involving $\sqrt{1-x^2}$ -type substitutions – restrict to $[-1, 1]$ explicitly.

10. Straight Lines

Key Results

- Distance point-line: $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$; distance between parallels: $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$
- Angle between lines: $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$; parallel: $m_1 = m_2$; perpendicular: $m_1 m_2 = -1$
- Foot of perpendicular / image of a point: $\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2 \frac{ax_1 + by_1 + c}{a^2 + b^2}$ (image); half that for foot
- Angle bisectors: $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$

⚡ TRAP / TRICK

Acute vs obtuse angle bisector: make constants c, c' positive first, then $aa' + bb' > 0 \Rightarrow$ "+" sign gives obtuse bisector – sign convention is the usual mistake.

Vertical lines have undefined slope – don't force $y = mx + c$ form; handle $x = k$ separately.

Reflection formula factor of -2 , not -1 – a very common silent error when finding the image of a point.

11. Circles

Key Results

- $x^2 + y^2 + 2gx + 2fy + c = 0$: centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$
- Tangent at (x_1, y_1) : $T = 0$, i.e. $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
- Length of tangent from (x_1, y_1) : $\sqrt{S_1}$, where $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$
- Condition tangency to line: perpendicular distance from centre = radius
- Family of circles through intersection: $S_1 + \lambda S_2 = 0$ or $S + \lambda L = 0$

⚡ TRAP / TRICK

$S_1 > 0, = 0, < 0$ tells whether the point is outside/on/inside the circle – use this instantly instead of computing full distance.

Two circles' radical axis $S_1 - S_2 = 0$ is a straight line only when both have the $x^2 + y^2$ coefficient equal to 1 (or equal to each other) – normalize first.

Orthogonality condition $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ is often mixed up with the tangency condition – keep them mentally separate.

12. Parabola, Ellipse, Hyperbola**Standard Forms ($y^2 = 4ax$ family)**

- Parabola $y^2 = 4ax$: focus $(a, 0)$, directrix $x = -a$, tangent (slope form) $y = mx + a/m$
- Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$): $b^2 = a^2(1 - e^2)$, foci $(\pm ae, 0)$, tangent $y = mx \pm \sqrt{a^2m^2 + b^2}$
- Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$: $b^2 = a^2(e^2 - 1)$, foci $(\pm ae, 0)$, tangent $y = mx \pm \sqrt{a^2m^2 - b^2}$
- Rectangular hyperbola $xy = c^2$: parametric $(ct, c/t)$, tangent at t : $x/t + ty = 2c$
- Chord of contact / pair of tangents: $T = 0$ and $SS_1 = T^2$ (works across all conics)

⚡ TRAP / TRICK

Check the tangency condition before using slope-form tangent – for the hyperbola it needs $a^2m^2 - b^2 \geq 0$ (real tangent), unlike the ellipse where it's always real.

Focal distance shortcuts: parabola $= x_1 + a$; ellipse $= a \pm ex_1$; use these instead of the distance formula to save real time.

Latus rectum length: parabola $4a$, ellipse/hyperbola $2b^2/a$ – don't confuse the ellipse/hyperbola form with $2a$.

13. Limits, Continuity & Differentiability**Standard Limits**

- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$
- $\lim_{x \rightarrow 0} (1+x)^{1/x} = e$; $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$
- $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$
- $1^\infty, 0^0, \infty^0$ handled via $\lim e^{[f(x)-1]g(x)}$ for $f \rightarrow 1, g \rightarrow \infty$
- Continuity at a : $\lim_{x \rightarrow a^-} f = \lim_{x \rightarrow a^+} f = f(a)$; differentiability requires LHD = RHD (and continuity first)

⚡ TRAP / TRICK

Differentiability $\Rightarrow$ continuity, not the converse – always check continuity first before testing LHD/RHD; a discontinuous function is automatically non-differentiable there.

1^∞ form is NOT equal to 1, it's indeterminate – never substitute directly; always use the exponential trick.

Piecewise/modulus functions: check differentiability strictly at the breakpoints only – elsewhere the function is smooth by each piece's own rule.

14. Differentiation**Rules**

- Product: $(fg)' = f'g + fg'$; Quotient: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$; Chain: $\frac{d}{dx} f(g(x)) = f'(g)g'$
- $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
- Parametric: $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$
- Implicit differentiation: differentiate both sides w.r.t. x , treat y as $y(x)$, collect dy/dx terms
- Logarithmic differentiation for $f(x)^{g(x)}$: take $\ln$ in both sides first

⚡ TRAP / TRICK

$f(x)^{g(x)}$ forms **MUST use log differentiation** – treating it as a simple power or exponential rule is one of the most common JEE errors.

Inverse trig substitutions ($x = \sin \theta$ etc.) require restricting θ to the correct principal branch before differentiating – else sign errors creep in.

Second derivative via parametric form is NOT $\frac{d^2y/d\theta^2}{d^2x/d\theta^2}$ – must differentiate dy/dx again w.r.t. θ then divide by $dx/d\theta$.

15. Application of Derivatives**Key Results**

- Tangent at (x_1, y_1) : $y - y_1 = f'(x_1)(x - x_1)$; Normal: $y - y_1 = -\frac{1}{f'(x_1)}(x - x_1)$
- Increasing/decreasing: $f'(x) > 0 \Rightarrow$ increasing; $f'(x) < 0 \Rightarrow$ decreasing (on the interval)
- Local max/min: $f'(x) = 0$ and sign change of f' (first derivative test) or $f''(x) < 0 / > 0$ (second derivative test)
- Rolle's theorem needs $f(a) = f(b)$; LMVT gives $f'(c) = \frac{f(b) - f(a)}{b - a}$, no equal-endpoint condition needed
- Rate of change (related rates): identify the fixed relation, differentiate w.r.t. t

⚡ TRAP / TRICK

Second derivative test fails when $f''(x) = 0$ – must fall back to first derivative test in that case, not conclude "no extremum."

Global (absolute) max/min on a closed interval: always check the endpoints too, not just critical points – easy marks lost here.

Rolle's theorem preconditions (continuity on $[a, b]$, derivability on (a, b) , $f(a) = f(b)$) must ALL hold – a common MCQ trap tests exactly which condition fails.

16. Indefinite Integration**Standard & Special Forms**

- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$; $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$
- $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$
- By parts (ILATE priority: Inverse, Log, Algebraic, Trig, Exponential – pick "first function" in that order)
- $\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$
- $\int \frac{px+q}{ax^2+bx+c} dx$: split into a multiple of $\frac{d}{dx}(ax^2+bx+c)$ plus a constant remainder

⚡ TRAP / TRICK

ILATE is a guide, not a law – occasionally reversing the choice simplifies faster (e.g. $\int x^2 e^x dx$ needs algebraic as "first function," consistent with ILATE, but always sanity-check after one step).

Forgetting $+c$ costs marks in subjective exams and can make an MCQ answer look "different but equivalent" – always verify by differentiating back if time allows.

Trig integrals of $\frac{dx}{a+b \sin x}$ type: use $t = \tan(x/2)$, NOT $t = \tan x$ – using the wrong substitution is a very frequent error.

17. Definite Integration**Properties**

- $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ – the single most useful JEE property
- $\int_0^{2a} f(x) dx = \int_0^a [f(x) + f(2a-x)] dx$
- $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ if f even, $= 0$ if f odd
- Periodic function of period T : $\int_0^{nT} f(x) dx = n \int_0^T f(x) dx$
- Leibniz rule: $\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) dt = h'(x)f(h(x)) - g'(x)f(g(x))$

⚡ TRAP / TRICK

King's property ($x \rightarrow a + b - x$) is the first thing to try when the integrand looks messy or when $f(x) + f(a + b - x)$ simplifies nicely – don't jump straight to brute-force integration.

Even/odd shortcut only applies to symmetric limits $[-a, a]$ – verify the limits before applying, a very common careless error.

Definite integral as a limit of a sum: match $\frac{1}{n} \sum f(r/n)$ to $\int_0^1 f(x) dx$ carefully – off-by-one in the summation range changes the limits of integration.

18. Area Under Curves & Differential Equations**Area**

- Area between $y = f(x)$ and x -axis on $[a, b]$: $\int_a^b |f(x)| dx$ – split at points where f changes sign
- Area between two curves: $\int_a^b [f(x) - g(x)] dx$ (upper – lower), or integrate w.r.t. y if curves are easier that way

Differential Equations

- Variable separable: collect x, y terms, integrate both sides
- Linear DE: $\frac{dy}{dx} + Py = Q$; I.F. = $e^{\int P dx}$; solution $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.} dx + c$
- Homogeneous DE ($\frac{dy}{dx} = f(y/x)$): substitute $y = vx$
- Order = highest derivative present; Degree = power of the highest-order derivative (after removing radicals/fractions)

⚡ TRAP / TRICK

Area sign: always take $|f(x)|$ or split the interval at roots of $f(x) = 0$ – a naive single integral can give a wrongly cancelled (too small) answer.

Degree is undefined if the DE cannot be written as a polynomial in derivatives (e.g. contains $\sin(dy/dx)$) – a favourite conceptual MCQ trap.

Linear DE integrating factor applies only when the equation is linear in y (or in x , treating x as the dependent variable) – check the form before computing I.F.

19. Vectors**Key Results**

- $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$; $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin \theta \hat{n}$
- Scalar triple product $[\vec{a} \vec{b} \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$; coplanar iff $[\vec{a} \vec{b} \vec{c}] = 0$
- Area of triangle = $\frac{1}{2} |\vec{AB} \times \vec{AC}|$; Volume of tetrahedron = $\frac{1}{6} |[\vec{AB} \vec{AC} \vec{AD}]|$
- Vector triple product: $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ ("BAC – CAB")
- Projection of $\vec{a}$ on $\vec{b}$: $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

⚡ TRAP / TRICK

$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$ in general – vector triple product is NOT associative; always check which bracket the question means.

Coplanarity test via scalar triple product = 0 is far faster than solving simultaneous equations – default to this method.

Unit vector direction: $\hat{n} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$ – remember both directions exist unless orientation is specified.

20. 3D Geometry**Lines & Planes**

- Direction cosines: $\ell^2 + m^2 + n^2 = 1$; angle between lines: $\cos \theta = |\ell_1 \ell_2 + m_1 m_2 + n_1 n_2|$
- Plane: $ax + by + cz + d = 0$; distance from point: $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$
- Angle between line and plane: $\sin \theta = \frac{|a\ell + bm + cn|}{\sqrt{a^2 + b^2 + c^2} \sqrt{\ell^2 + m^2 + n^2}}$
- Shortest distance between skew lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$, $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$: $\frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$
- Family of planes through intersection of two planes: $P_1 + \lambda P_2 = 0$

⚡ TRAP / TRICK

Angle between line & plane uses sin, angle between two lines/two planes uses cos – mixing these up is extremely common under time pressure.

Skew-line test: lines are skew only if they are NOT parallel AND shortest distance $\neq 0$ – check both conditions, not just one.

Foot of perpendicular from a point to a plane: parametrize along the normal direction (a, b, c) , substitute into the plane equation to find the parameter – faster than solving 3 equations directly.

21. Statistics**Measures**

- Mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$; Variance $\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} = \frac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2$
- Combined mean of two groups: $\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$
- Effect of scaling: if $y_i = ax_i + b$, then $\bar{y} = a\bar{x} + b$, but $\sigma_y = |a|\sigma_x$ (variance unaffected by b)

⚡ TRAP / TRICK

Adding a constant to every observation does not change variance – only scaling (multiplying) changes it, by a^2 for variance and $|a|$ for SD; a frequent trick-question setup.

Combined variance formula is NOT just the average of the two variances – it must also include the $(\bar{x}_1 - \bar{x})^2$ correction terms.