

SIMPLIFIED EDUCATION

First-Principles Learning · JEE · Olympiads · Foundation

CALCULUS

Complete Revision Compendium

Formulae · Derivation Logic · JEE Tricks · Common Traps

For JEE Main & Advanced

Compiled by

Sunil Yadav

14 Years of Teaching Experience

sunilyadav.org

How to Use This Guide

This compendium is built around a simple belief: **formulae stick when you understand where they come from, not when you memorise them in isolation.** Each topic gives you:

- The **core formula block** – everything you need at a glance.
- **Why it works** – a one-line first-principles anchor so the formula is rebuildable even if you forget it under pressure.
- **JEE Trick** boxes (gold) – shortcuts, pattern-recognition, and time-saving substitutions that actually show up in JEE Main/Advanced.
- **Common Mistake** boxes (red) – the exact traps that cost marks every single year.

Revision strategy: On your first pass, read the “Why it works” lines carefully. On every pass after that, skim only the formula boxes and trick boxes – that’s your 15-minute-before-exam version.

Contents

How to Use This Guide	1
Limits	3
Standard Limit Formulae	3
L’Hospital’s Rule	3
Squeeze (Sandwich) Theorem	3
Series Expansions Worth Memorising	4
Left-Hand and Right-Hand Limits	4
Continuity and Differentiability	4
Continuity	4
Differentiability	5
Algebra of Continuous/Differentiable Functions	5
Differentiation	5
First Principles	5
Standard Derivatives	5
Rules of Differentiation	6
Second-Order & Higher Derivatives in Parametric Form	6
Differentiation of Inverse Functions	6
Useful Substitution Tricks for Inverse Trig Differentiation	7
Rolle’s Theorem and Lagrange’s Mean Value Theorem (LMVT)	7
Application of Derivatives	7
Tangents and Normals	8
Rate of Change (Related Rates)	8
Increasing and Decreasing Functions (Monotonicity)	8
Maxima and Minima	9
Concavity and Points of Inflection	9
Indefinite Integration	9
Standard Integrals	9
Standard Integrals – Algebraic Forms (Memorise Cold)	10
Integration by Substitution – Pattern Bank	10
Integration by Parts	10

Partial Fractions for Rational Functions	11
Reduction Formulae (JEE Advanced Favorites)	11
Definite Integration	11
Fundamental Theorem of Calculus	11
Properties of Definite Integrals (Memorise – These ARE the JEE Tricks)	12
Definite Integral as Limit of a Sum	12
Leibniz Rule (Differentiation Under the Integral Sign)	13
Walli's Formula (Direct Result – No Derivation Needed in Exam)	13
Estimating Definite Integrals & Inequalities	13
Area Under Curves	14
Differential Equations	14
Order, Degree, and Formation	14
Variable Separable	14
Homogeneous Equations	14
Linear Differential Equation (First Order)	15
Quick Identification Table	15
JEE Exam Strategy – Calculus-Specific	15
Time-Saving Decision Tree for Integration	15
Highest-Yield Topics by Frequency (Historical JEE Pattern)	16
Common Silly Mistakes That Cost Marks	16
Final Revision Checklist	16

Limits

Standard Limit Formulae

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{\sin x}{x} &= 1 & \lim_{x \rightarrow 0} \frac{\tan x}{x} &= 1 & \lim_{x \rightarrow 0} \frac{x}{\sin x} &= 1 \\
 \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} &= 1 & \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} &= 1 & \\
 \lim_{x \rightarrow 0} \frac{e^x - 1}{x} &= 1 & \lim_{x \rightarrow 0} \frac{a^x - 1}{x} &= \ln a & \lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} &= 1 \\
 \lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x} &= n & \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x &= e & \lim_{x \rightarrow 0} (1+x)^{1/x} &= e \\
 \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} &= na^{n-1} & \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} &= \frac{1}{2} &
 \end{aligned}$$

Why it works: Every one of these comes from a Taylor expansion truncated at first order. $\sin x \approx x - \frac{x^3}{6}$, $e^x \approx 1 + x + \frac{x^2}{2}$, $\ln(1+x) \approx x - \frac{x^2}{2}$, $(1+x)^n \approx 1 + nx$. If you remember the expansions, you never need to memorise the limit list separately – you can rebuild any of them, including ones not listed here.

L'Hospital's Rule

If $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ or $\pm\infty$ (i.e. form is $\frac{0}{0}$ or $\frac{\infty}{\infty}$), then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

provided the RHS limit exists. Can be applied repeatedly.

★ JEE TRICK / SHORTCUT

Convert before differentiating. $0 \cdot \infty$ forms: rewrite as $\frac{f}{1/g}$ to get $\frac{0}{0}$.

$\infty - \infty$ forms: combine into a single fraction first.

$1^\infty, 0^0, \infty^0$ forms: take $\ln$ of the expression, reduce to $0 \cdot \infty$, solve, then exponentiate back.

Speed tip for JEE Main: for 1^∞ type limits $\lim_{x \rightarrow a} (f(x))^{g(x)}$ where $f \rightarrow 1$, use the direct formula

$$\lim_{x \rightarrow a} f(x)^{g(x)} = e^{\lim_{x \rightarrow a} g(x)[f(x)-1]}$$

This skips the log-and-expand process entirely and is the fastest route to MCQ answers.

! COMMON MISTAKE

L'Hospital only applies to $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Applying it to forms like $\frac{1}{0}$ or a determinate limit is a very common scoring error – always check the form first by direct substitution.

Squeeze (Sandwich) Theorem

If $g(x) \leq f(x) \leq h(x)$ near $x = a$ and $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} f(x) = L$.

★ JEE TRICK / SHORTCUT

Used heavily for limits involving $[x]$ (greatest integer), $\{x\}$ (fractional part), or bounded oscillating terms like $\sin(1/x)$ multiplied by something $\rightarrow 0$. If you see $x \sin(1/x)$ as $x \rightarrow 0$, immediately bound $-1 \leq \sin(1/x) \leq 1$ and squeeze – don't try L'Hospital, it fails here since $\sin(1/x)$ has no limit.

Series Expansions Worth Memorising

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \quad (-1 < x \leq 1) \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \\ \tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots \\ (1+x)^n &= 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots \end{aligned}$$

★ JEE TRICK / SHORTCUT

For limits of the form $\lim_{x \rightarrow 0} \frac{\text{combination of } \sin, \cos, e^x, \ln(1+x)}{x^k}$, series expansion is often faster and safer than repeated L'Hospital, especially in JEE Advanced where the function is a sum of several terms. Expand each term to the order matching the denominator's power, cancel, and read off the answer – no differentiation needed.

Left-Hand and Right-Hand Limits

$$\lim_{x \rightarrow a^-} f(x) = \lim_{h \rightarrow 0^+} f(a-h) \quad \lim_{x \rightarrow a^+} f(x) = \lim_{h \rightarrow 0^+} f(a+h)$$

Limit exists at $x = a \iff \text{LHL} = \text{RHL}.$

★ JEE TRICK / SHORTCUT

For functions with $[x]$, $|x|$, or piecewise definitions, ALWAYS check LHL and RHL separately near the breakpoint – never assume direct substitution works. For limits at $x \rightarrow \infty$, divide numerator and denominator by the highest power of x rather than direct substitution.

Continuity and Differentiability

Continuity

$f(x)$ is continuous at $x = a$ if

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

Why it works: Continuity is just “no surprise jump.” All three quantities – the value the function is heading to from the left, from the right, and the actual value sitting at the point – must agree.

Differentiability

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

exists only if LHD = RHD:

$$\text{LHD} = \lim_{h \rightarrow 0^-} \frac{f(a+h) - f(a)}{h}, \quad \text{RHD} = \lim_{h \rightarrow 0^+} \frac{f(a+h) - f(a)}{h}$$

Key theorem: Differentiability $\implies$ Continuity.

(Converse is FALSE – e.g. $f(x) = |x|$ is continuous but not differentiable at 0.)

★ JEE TRICK / SHORTCUT

Standard non-differentiable points to scan for instantly: $|x|$, $[x]$, $\{x\}$ at integers, max/min of two functions at their intersection, and any modulus-wrapped expression at the point where the inside becomes zero. In JEE Advanced, the moment you see $|g(x)|$ or $|x - a|$ inside a function, check differentiability at the zero of $g(x)$ first – that's almost always where the question is testing you.

! COMMON MISTAKE

A function can be continuous everywhere yet differentiable nowhere (Weierstrass-type pathological functions) – but for JEE purposes, the practical trap is students assuming continuity automatically gives differentiability. It never does. Always verify LHD = RHD explicitly at suspicious points.

Algebra of Continuous/Differentiable Functions

If f, g are continuous (or differentiable) at $x = a$, so are $f \pm g$, fg , and f/g (provided $g(a) \neq 0$).

Composition of continuous functions is continuous. Composition of differentiable functions is differentiable (chain rule applies).

★ JEE TRICK / SHORTCUT

Product trick: if f is differentiable but g is not differentiable at a , the product fg can still be differentiable at a if $f(a) = 0$. This exact scenario (e.g. $f(x) = x^2$, $g(x) = |x|$ at $x = 0$) appears repeatedly in JEE – always test it by checking whether the non-differentiable part gets multiplied by zero.

Differentiation

First Principles

$$\frac{d}{dx}f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Why it works: Every derivative formula below is just this limit, evaluated once and for all for a given function family. If you ever blank out on a derivative under exam pressure, you can always fall back to this definition for simple functions like x^n , $\sin x$, or e^x .

Standard Derivatives

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(\sin x) = \cos x$		
$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(a^x) = a^x \ln a$	$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{ x \sqrt{x^2-1}}$	$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{ x \sqrt{x^2-1}}$
$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$	$\frac{d}{dx}(\sec x) = \sec x \tan x$		
	$\frac{d}{dx}(\csc x) = -\csc x \cot x$		

Rules of Differentiation

Product Rule: $(uv)' = u'v + uv'$

Quotient Rule: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Implicit Differentiation: differentiate both sides w.r.t. x , treating y as $y(x)$, then solve for $\frac{dy}{dx}$.

Parametric form: if $x = f(t)$, $y = g(t)$, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Logarithmic differentiation: for $y = [f(x)]^{g(x)}$, take $\ln$ both sides: $\ln y = g(x) \ln f(x)$, then differentiate.

★ JEE TRICK / SHORTCUT

Whenever you see a variable in the exponent of another variable (x^x , $x^{\sin x}$, $(\sin x)^x$, etc.), logarithmic differentiation is mandatory – standard power/exponential rules don't directly apply.

For products of 3+ functions like $y = uvw$, don't expand and use product rule repeatedly. Instead take $\ln y = \ln u + \ln v + \ln w$ and differentiate – much faster and far less error-prone in JEE time pressure.

Second-Order & Higher Derivatives in Parametric Form

If $x = f(t)$, $y = g(t)$:

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

NOT $\frac{d^2y/dt^2}{d^2x/dt^2}$ – this is the single most common error in parametric second derivatives.

! COMMON MISTAKE

Students very frequently write $\frac{d^2y}{dx^2} = \frac{y''(t)}{x''(t)}$. This is wrong. You must differentiate $\frac{dy}{dx}$ (itself a function of t) with respect to t again, then divide by $\frac{dx}{dt}$ once more.

Differentiation of Inverse Functions

If $y = f(x)$ is invertible and differentiable with $f'(x) \neq 0$, and $x = f^{-1}(y)$, then

$$\frac{d}{dy} [f^{-1}(y)] = \frac{1}{f'(x)} \quad \text{evaluated at } x = f^{-1}(y)$$

Useful Substitution Tricks for Inverse Trig Differentiation

★ JEE TRICK / SHORTCUT

These substitutions convert messy inverse-trig expressions into simple linear angle expressions before differentiating – always substitute **FIRST**, simplify, **THEN** differentiate.

- $\sqrt{1-x^2}$ present $\Rightarrow$ put $x = \sin \theta$ or $x = \cos \theta$
- $\sqrt{1+x^2}$ present $\Rightarrow$ put $x = \tan \theta$
- $\sqrt{x^2-1}$ present $\Rightarrow$ put $x = \sec \theta$
- $\frac{2x}{1+x^2}$ or $\frac{1-x^2}{1+x^2}$ patterns $\Rightarrow$ put $x = \tan \theta$ (these are double-angle disguises: $\sin 2\theta$, $\cos 2\theta$)
- $\sqrt{\frac{1-\cos x}{1+\cos x}}$ type $\Rightarrow$ this is $\tan(x/2)$ in disguise

Why it works: Functions like $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ look intimidating until you recognise $\frac{2x}{1+x^2} = \sin(2\theta)$ when $x = \tan \theta$. The whole expression collapses to $y = 2\theta = 2\tan^{-1}x$, which differentiates in one line. JEE deliberately disguises simple derivatives behind trig identities – recognising the disguise IS the skill being tested.

Rolle's Theorem and Lagrange's Mean Value Theorem (LMVT)

Rolle's Theorem: If f is continuous on $[a, b]$, differentiable on (a, b) , and $f(a) = f(b)$, then there exists $c \in (a, b)$ such that $f'(c) = 0$.

LMVT: If f is continuous on $[a, b]$ and differentiable on (a, b) , then there exists $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

★ JEE TRICK / SHORTCUT

LMVT/Rolle's questions in JEE almost always reduce to: "show some equation has a root in an interval."

Strategy: construct an auxiliary function $g(x)$ such that $g'(x)$ equals the expression you're asked to prove has a root, then apply Rolle's on g . E.g. to show $f'(x) = c$ has a solution, you may need to define $g(x) = f(x) - cx$ and apply Rolle's to g .

! COMMON MISTAKE

Both theorems strictly require continuity on the *closed* interval and differentiability on the *open* interval. If a function fails to be differentiable at even one interior point, Rolle's/LMVT cannot be applied – this is a frequent "assumption violated" trap in Advanced-level reasoning questions.

Application of Derivatives

Tangents and Normals

At point (x_1, y_1) on curve $y = f(x)$ with slope $m = f'(x_1)$:

Tangent: $y - y_1 = m(x - x_1)$ **Normal:** $y - y_1 = -\frac{1}{m}(x - x_1)$

Angle between two curves at intersection point: if slopes are m_1, m_2 ,

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Curves are **orthogonal** if $m_1 m_2 = -1$.

★ JEE TRICK / SHORTCUT

Length of tangent, normal, sub-tangent, sub-normal (distance measured along the x -axis or the line itself) are direct-formula questions in JEE Main:

- Length of tangent = $\left| \frac{y_1 \sqrt{1 + m^2}}{m} \right|$
- Length of normal = $|y_1 \sqrt{1 + m^2}|$
- Length of sub-tangent = $\left| \frac{y_1}{m} \right|$
- Length of sub-normal = $|y_1 m|$

Memorise these four directly – deriving them each time wastes precious exam minutes.

Rate of Change (Related Rates)

If two quantities x, y are related by an equation and both vary with time t , differentiate the relation w.r.t. t using the chain rule to connect $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

★ JEE TRICK / SHORTCUT

Always write the governing equation first (e.g. Volume of a cone, Pythagoras for a ladder problem) before differentiating. Identify what's constant (often a fixed length) and what's changing – substitute known numeric values *only after* differentiating, never before, or you'll lose a variable you needed.

Increasing and Decreasing Functions (Monotonicity)

f is **increasing** on (a, b) if $f'(x) \geq 0 \forall x \in (a, b)$ (strictly increasing if $f'(x) > 0$).
 f is **decreasing** on (a, b) if $f'(x) \leq 0 \forall x \in (a, b)$.

★ JEE TRICK / SHORTCUT

Sign-scheme method: find $f'(x)$, factor it completely, mark all roots on the number line, and test the sign of each factor in each interval. This single method handles 90% of monotonicity, maxima/minima, and inequality-proving questions in JEE.

Proving inequalities trick: to prove $f(x) > g(x)$ for $x > a$, define $h(x) = f(x) - g(x)$, show $h(a) = 0$ (or $h(a) \geq 0$) and $h'(x) > 0$ for $x > a$ – this proves $h(x) > 0$, i.e. the inequality, without ever solving for x directly.

Maxima and Minima

First Derivative Test: at a critical point c where $f'(c) = 0$:

f' changes $+$ $\rightarrow$ $- \Rightarrow$ local max; f' changes $- \rightarrow + \Rightarrow$ local min; no change $\Rightarrow$ neither.

Second Derivative Test: at critical point c with $f'(c) = 0$:

$f''(c) < 0 \Rightarrow$ local max; $f''(c) > 0 \Rightarrow$ local min; $f''(c) = 0 \Rightarrow$ test inconclusive (use higher derivatives or first derivative test).

Global max/min on $[a, b]$: compare $f(a)$, $f(b)$, and $f(c)$ at all critical points $c \in (a, b)$.

★ JEE TRICK / SHORTCUT

Optimization word-problems (JEE Main staple): 1) Identify the quantity to optimise and write it as a function of ONE variable using the constraint to eliminate others. 2) Differentiate, set to zero, solve. 3) Confirm max/min using second derivative test. 4) Check boundary values if the domain is closed – the optimum could be at an endpoint, not an interior critical point.

AM-GM shortcut: many JEE optimization problems (minimise $x + \frac{a}{x}$ type expressions) can be solved instantly via $AM \geq GM$ without calculus at all: $x + \frac{a}{x} \geq 2\sqrt{a}$, with equality at $x = \sqrt{a}$. If you spot this form, skip differentiation entirely.

! COMMON MISTAKE

$f''(c) = 0$ does NOT mean "no extremum" – it means the second derivative test is inconclusive, and you must fall back to the first derivative test or check higher-order derivatives. Many students incorrectly conclude "neither max nor min" the moment they see $f''(c) = 0$.

Concavity and Points of Inflection

$f''(x) > 0$ on an interval $\Rightarrow$ concave up (convex). $f''(x) < 0 \Rightarrow$ concave down.

A point of inflection occurs where $f''(x) = 0$ **and** f'' changes sign across that point.

Indefinite Integration

Standard Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int e^x dx = e^x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \tan x dx = \ln |\sec x| + C$$

$$\int \sec x dx = \ln |\sec x + \tan x| + C$$

$$\int \frac{1}{x} dx = \ln |x| + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \cot x dx = \ln |\sin x| + C$$

$$\int \csc x dx = \ln |\csc x - \cot x| + C$$

Standard Integrals – Algebraic Forms (Memorise Cold)

$$\begin{aligned}\int \frac{dx}{x^2 + a^2} &= \frac{1}{a} \tan^{-1} \frac{x}{a} + C \\ \int \frac{dx}{x^2 - a^2} &= \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C \\ \int \frac{dx}{a^2 - x^2} &= \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C \\ \int \frac{dx}{\sqrt{a^2 - x^2}} &= \sin^{-1} \frac{x}{a} + C \\ \int \frac{dx}{\sqrt{x^2 + a^2}} &= \ln \left| x + \sqrt{x^2 + a^2} \right| + C \\ \int \frac{dx}{\sqrt{x^2 - a^2}} &= \ln \left| x + \sqrt{x^2 - a^2} \right| + C \\ \int \sqrt{a^2 - x^2} dx &= \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C \\ \int \sqrt{x^2 + a^2} dx &= \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln \left| x + \sqrt{x^2 + a^2} \right| + C \\ \int \sqrt{x^2 - a^2} dx &= \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln \left| x + \sqrt{x^2 - a^2} \right| + C\end{aligned}$$

★ JEE TRICK / SHORTCUT

These nine algebraic forms cover almost every "irreducible quadratic under a root" question. **Pattern recognition:** look at the sign pattern under the root/denominator – $(a^2 - x^2)$, $(x^2 + a^2)$, or $(x^2 - a^2)$ – and match directly to the table instead of re-deriving via substitution every time.

Integration by Substitution – Pattern Bank

★ JEE TRICK / SHORTCUT

Scan the integrand for these patterns *before* trying anything else – substitution should be your first instinct, not your last resort:

- $\int f'(x) [f(x)]^n dx \Rightarrow$ put $t = f(x)$
- $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$ (spot this instantly – numerator is the exact derivative of denominator)
- $\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$
- Integrand contains $\sqrt{a^2 - x^2} \Rightarrow x = a \sin \theta$; $\sqrt{a^2 + x^2} \Rightarrow x = a \tan \theta$; $\sqrt{x^2 - a^2} \Rightarrow x = a \sec \theta$
- $\int \frac{dx}{x\sqrt{x^{2n} \pm a^2}}$ type $\Rightarrow$ substitute $t = 1/x$
- Rational function of $\sin x, \cos x$ only $\Rightarrow$ universal substitution $t = \tan(x/2)$ converts everything to a rational function of t

Integration by Parts

$$\int u v dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx$$

ILATE rule for choosing u (first function, the one you differentiate):

Inverse trig $\rightarrow$ Logarithmic $\rightarrow$ Algebraic $\rightarrow$ Trigonometric $\rightarrow$ Exponential

Whichever appears earlier in ILATE is chosen as u .

★ JEE TRICK / SHORTCUT

The " $\int e^{ax} \sin(bx)$ trap": integration by parts here never terminates by itself – you must apply it twice and then solve algebraically for the original integral I (it reappears on the RHS). Standard result:

$$\int e^{ax} \sin(bx) dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$$

Recognise this form immediately rather than re-deriving it in the exam.

Partial Fractions for Rational Functions

For $\frac{P(x)}{Q(x)}$ with $\deg P < \deg Q$, decompose based on the factors of $Q(x)$:

Linear factor $(x - a) \rightarrow$ term $\frac{A}{x - a}$ Repeated $(x - a)^2 \rightarrow \frac{A}{x - a} + \frac{B}{(x - a)^2}$

Irreducible quadratic $(x^2 + bx + c) \rightarrow$ term $\frac{Ax + B}{x^2 + bx + c}$

★ JEE TRICK / SHORTCUT

If $\deg P \geq \deg Q$, divide first (polynomial long division) to get a polynomial + proper-fraction remainder, THEN apply partial fractions to the remainder only.

Reduction Formulae (JEE Advanced Favorites)

$$\int \sin^n x dx : \quad I_n = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} I_{n-2}$$

$$\int \tan^n x dx : \quad I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$$

★ JEE TRICK / SHORTCUT

For odd powers of $\sin x$ or $\cos x$: split off one factor and substitute using the Pythagorean identity (e.g. $\sin^3 x = \sin x(1 - \cos^2 x)$, then let $t = \cos x$).

For even powers: use power-reduction identities $\sin^2 x = \frac{1 - \cos 2x}{2}$, $\cos^2 x = \frac{1 + \cos 2x}{2}$ repeatedly rather than reduction formulae – it's faster by hand.

Definite Integration

Fundamental Theorem of Calculus

If $F'(x) = f(x)$, then $\int_a^b f(x) dx = F(b) - F(a)$.

Properties of Definite Integrals (Memorise – These ARE the JEE Tricks)

P1: $\int_a^b f(x) dx = \int_a^b f(t) dt$ (dummy variable doesn't matter)

P2: $\int_a^b f(x) dx = - \int_b^a f(x) dx$

P3: $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

P4 (King's Property): $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

P5: $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ (special case of P4)

P6: $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$

P7: $\int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx & \text{if } f \text{ is even} \\ 0 & \text{if } f \text{ is odd} \end{cases}$

P8 (Periodicity): If $f(x+T) = f(x)$, then $\int_0^{nT} f(x) dx = n \int_0^T f(x) dx$, and $\int_a^{a+T} f(x) dx$ is independent of a .

★ JEE TRICK / SHORTCUT

P4 (King's Property) solves more JEE definite-integral problems than any other single tool. Whenever you see an integral with limits $[a, b]$ and a function involving $\sin x$, $\cos x$, or a fraction that looks symmetric, try replacing $x \rightarrow a+b-x$. Common signal: integrals of the form $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x) + f(\cos x)} dx$ – applying King's property and adding the original integral to the transformed one collapses everything to $\frac{\pi}{4}$ *without ever finding the antiderivative*.

★ JEE TRICK / SHORTCUT

The "Add the King's Property version to itself" master trick: If $I = \int_a^b f(x) dx$ and after applying P4 you get $I = \int_a^b g(x) dx$ where $f(x) + g(x)$ simplifies nicely (often to a constant), then

$$2I = \int_a^b [f(x) + g(x)] dx \implies I = \frac{1}{2} \int_a^b [f(x) + g(x)] dx$$

This is THE go-to move for trig-heavy definite integrals with symmetric limits like $[0, \pi]$ or $[0, \pi/2]$.

! COMMON MISTAKE

P7 (odd/even shortcut) only works for symmetric limits $[-a, a]$. Don't try to force-fit it onto asymmetric limits like $[-a, 2a]$ – split the integral at 0 first if you want to use odd/even properties on part of it.

Definite Integral as Limit of a Sum

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a + rh), \quad h = \frac{b-a}{n}$$

Special case ($a = 0, b = 1$):

$$\int_0^1 f(x) dx = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=0}^{n-1} f\left(\frac{r}{n}\right) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r}{n}\right)$$

★ JEE TRICK / SHORTCUT

When asked to evaluate $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 + r^2}$ or similar sums, the trick is to manufacture a $\frac{1}{n}$ outside and a $\frac{r}{n}$ inside that becomes your variable x . Rewrite the sum as $\frac{1}{n} \sum f(r/n)$, identify $f(x)$, and convert directly to $\int_0^1 f(x) dx$.

Leibniz Rule (Differentiation Under the Integral Sign)

If $F(x) = \int_{g(x)}^{h(x)} f(t) dt$, then

$$F'(x) = f(h(x)) \cdot h'(x) - f(g(x)) \cdot g'(x)$$

★ JEE TRICK / SHORTCUT

This appears in JEE Advanced disguised as "find $\frac{d}{dx}$ of an integral with variable limits." Don't try to evaluate the integral first – apply Leibniz directly. If the integrand itself also contains x (not just the limits), you need the full Leibniz integral rule with a partial-derivative term, which is rare at JEE level but worth knowing exists.

Walli's Formula (Direct Result – No Derivation Needed in Exam)

$$\int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx = \begin{cases} \frac{(n-1)(n-3) \cdots 1}{n(n-2) \cdots 2} \cdot \frac{\pi}{2} & n \text{ even} \\ \frac{(n-1)(n-3) \cdots 2}{n(n-2) \cdots 3} & n \text{ odd} \end{cases}$$

★ JEE TRICK / SHORTCUT

For mixed powers $\int_0^{\pi/2} \sin^m x \cos^n x dx$, the extended Walli's formula multiplies the two descending double-factorial chains together, with an extra $\frac{\pi}{2}$ factor only if *both* m and n are even. This single formula instantly solves what would otherwise take five lines of reduction formula work – worth memorising cold for JEE Advanced.

Estimating Definite Integrals & Inequalities

★ JEE TRICK / SHORTCUT

If $m \leq f(x) \leq M$ for all $x \in [a, b]$, then $m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$. JEE sometimes asks you to bound an integral rather than evaluate it exactly – recognise when the question wants an inequality/range answer instead of a numeric value (often signalled by options that are ranges, not single numbers).

Area Under Curves

Area between curve $y = f(x)$ and the x -axis from $x = a$ to $x = b$ (where $f(x) \geq 0$):

$$A = \int_a^b f(x) dx$$

Area between two curves $y = f(x)$ (upper) and $y = g(x)$ (lower) from $x = a$ to $x = b$:

$$A = \int_a^b [f(x) - g(x)] dx$$

Area with respect to y -axis (when easier): $A = \int_c^d [x_{\text{right}} - x_{\text{left}}] dy$

★ JEE TRICK / SHORTCUT

Step-by-step area strategy: 1) Sketch the curves roughly (even a rough sketch prevents sign errors). 2) Find intersection points by equating the two functions – these become your limits. 3) Identify which curve is on top in each sub-interval (it can switch – if it does, split the integral at the crossover point). 4) Integrate [upper – lower], never $|f - g|$ blindly without checking the sign in each region.

When the curve is below the axis, the definite integral gives a negative value, but area is always reported as positive – take the absolute value, or better, identify the sign-changing point and split the integral there.

! COMMON MISTAKE

A very common mistake: integrating w.r.t. x when the region is naturally bounded by horizontal lines (easier w.r.t. y), leading to needlessly split integrals. If the curves are more naturally described as $x = f(y)$ (e.g. parabolas opening sideways, or when the upper/lower boundary switches frequently in x but not in y), switch the entire setup to integrate w.r.t. y .

Differential Equations

Order, Degree, and Formation

Order: highest derivative present. **Degree:** power of the highest-order derivative (after the equation is made polynomial in derivatives).

Formation: eliminate arbitrary constants from a given family of curves by differentiating as many times as there are constants, then combining.

Variable Separable

If the ODE can be written as $f(y) dy = g(x) dx$, integrate both sides directly:

$$\int f(y) dy = \int g(x) dx + C$$

Homogeneous Equations

If $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ (a function of y/x only), substitute $y = vx$, so $\frac{dy}{dx} = v + x \frac{dv}{dx}$. This converts the equation into a variable-separable one in v and x .

★ JEE TRICK / SHORTCUT

Recognition rule: if every term in $\frac{dy}{dx} = \frac{P(x,y)}{Q(x,y)}$ has P and Q as homogeneous polynomials of the *same degree* in x and y , it's homogeneous – use $y = vx$ immediately without trying other methods first.

Linear Differential Equation (First Order)

Standard form: $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x only.

Integrating Factor: $IF = e^{\int P dx}$

Solution: $y \cdot IF = \int Q \cdot IF dx + C$

★ JEE TRICK / SHORTCUT

If the equation is linear in x instead (form: $\frac{dx}{dy} + P_1x = Q_1$, with P_1, Q_1 functions of y), use $IF = e^{\int P_1 dy}$ and solve symmetrically with $x \cdot IF = \int Q_1 \cdot IF dy + C$. Many students only check for "linear in y " and miss this mirror case – always check both directions if the obvious one fails.

! COMMON MISTAKE

Before applying the IF formula, the coefficient of $\frac{dy}{dx}$ MUST be exactly 1. If your equation looks like $k(x)\frac{dy}{dx} + P(x)y = Q(x)$, divide through by $k(x)$ first – skipping this step is a very common reason for getting the wrong integrating factor.

Quick Identification Table

Pattern	Method
Separable into $f(y)dy = g(x)dx$	Direct integration
$\frac{dy}{dx} = F(y/x)$, homogeneous degree	$y = vx$ substitution
$\frac{dy}{dx} + Py = Q$ (linear in y)	Integrating factor $e^{\int P dx}$
$\frac{dx}{dy} + P_1x = Q_1$ (linear in x)	Integrating factor $e^{\int P_1 dy}$

JEE Exam Strategy – Calculus-Specific

Time-Saving Decision Tree for Integration

★ JEE TRICK / SHORTCUT

When you see an integral and don't immediately know the method, run through this checklist in order – it covers the method-selection logic that experienced solvers do almost subconsciously:

1. Is the numerator the derivative of the denominator (or close to it after a constant)? $\rightarrow \ln|f(x)|$ directly.
2. Does the integrand match a standard algebraic form ($\frac{1}{\sqrt{a^2-x^2}}$ etc.)? $\rightarrow$ direct formula.
3. Is it a product of an algebraic/trig/exponential function and a clearly "simpler-after-differentiating" function? $\rightarrow$ integration by parts (ILATE).
4. Is it a rational function $P(x)/Q(x)$ with factorable denominator? $\rightarrow$ partial fractions.
5. Does a part of the integrand look like $f'(x)$ times something in terms of $f(x)$? $\rightarrow$ substitution $t = f(x)$.

6. Rational function purely of $\sin x, \cos x$? $\rightarrow t = \tan(x/2)$ universal substitution.
7. None of the above and it's a definite integral with special limits (0 to $\pi/2$, $-a$ to a , etc.)? $\rightarrow$ try the properties (King's property, odd/even) before attempting to find the antiderivative at all.

Highest-Yield Topics by Frequency (Historical JEE Pattern)

Topic	What to drill
Definite integral properties	King's property, odd/even, periodicity – these alone solve a large fraction of integral-based questions without antiderivatives
Maxima/minima word problems	AM-GM shortcut first, calculus method as backup
Continuity/differentiability at a point	$ x $, $[x]$, piecewise junctions – check LHL/RHL and LHD/RHD explicitly
Area between curves	Always sketch; identify intersection points before integrating
Tangent/normal & angle between curves	Memorise the four length formulae directly
Differential equations	Recognise the type (separable / homogeneous / linear) in under 10 seconds

Common Silly Mistakes That Cost Marks

! COMMON MISTAKE

- Forgetting "+C" in indefinite integrals (auto-deducted in subjective papers, irrelevant in MCQ but signals carelessness).
- Sign errors when applying $\int_a^b f(x) dx = -\int_b^a f(x) dx$ inside a longer derivation.
- Dropping the modulus in $\ln|x|$, $\sec^{-1}$ domain, or $\sqrt{x^2} = |x|$ – especially dangerous in definite integrals where the sign affects the final numeric answer.
- Using L'Hospital on a non- $\frac{0}{0}$, non- $\frac{\infty}{\infty}$ form.
- Differentiating parametric second derivatives as $\frac{y''(t)}{x''(t)}$ instead of the correct two-step process.
- Concluding "no extremum" the instant $f''(c) = 0$ without falling back to the first derivative test.
- Applying Rolle's/LMVT when differentiability fails at even a single interior point.

Final Revision Checklist

Before walking into the exam, you should be able to do all of the following *without* looking anything up:

- Write all standard limits and the Taylor expansions they come from.
- State and apply L'Hospital's rule correctly with its conditions.
- Differentiate every standard function including all six inverse trig functions.
- Recognise and apply at least four of the inverse-trig substitution disguises.
- Apply the first and second derivative tests, and know when each fails.
- Recall all nine algebraic standard integrals from memory.

- State and apply King's property and the odd/even property of definite integrals.
- Identify the type of a first-order differential equation in seconds and recall its corresponding solution method.

Compiled by **Sunil Yadav** – Simplified Education
sunilyadav.org
First-principles teaching for JEE, Olympiads & Foundation